Verification — King-Post Roof Truss

FEM Calculator core solver checked against an independently derived method-of-joints solution

Structure type	Statically determinate pin-jointed roof truss (5 members, 4 joints)
Verification method	Exact method-of-joints solution (derived from first principles)
Quantities checked	Support reactions and axial force in every member — including a zero-force member
Result	**Exact match** — FEM agrees with method of joints to $\approx 10^{-12}$ (floating-point round-off only)

1. Structure and loading

A king-post roof truss spanning 6 m: pin support at A, roller at B, a bottom-chord (tie beam) joint at C (midspan), and the ridge apex at D, 2 m above C. Five pin-jointed members: the tie beam (AC, CB), the two rafters (AD, DB), and the king post (CD). A single vertical load P = 12 kN is applied at the ridge D — the load path a ridge purlin would impose. Every member is idealized as a pure two-force axial member (no bending) — exactly matching truss theory's assumptions.

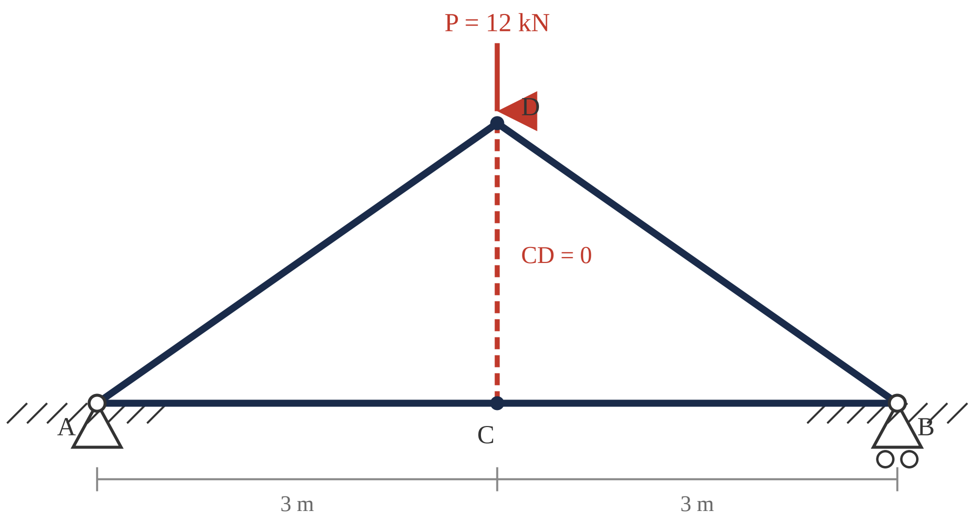

Fig. 1 — King-post truss geometry, supports, ridge load. CD (dashed) is the zero-force king post.

2. Hand solution (method of joints)

By symmetry, the ridge load splits evenly between the two supports:

> Reactions: $A_x = 0$, $A_y = B_y = P/2 =$ **6 kN**
>
> Joint C (no external load; AC, CB horizontal): $\Sigma F_y = 0 \rightarrow T_{CD} = \mathbf{0}$ — the king post is a *zero-force member* for this loading
>
> Joint D: $\Sigma F_x = 0 \rightarrow T_{AD} = T_{DB} = T_r$; $\Sigma F_y = 0 \rightarrow -2\cdot(2/\sqrt{13})\cdot T_r - T_{CD} = 12 \rightarrow T_r = -3\sqrt{13} \approx \mathbf{-10.8167\ kN}$ (compression)
>
> Joint A: $\Sigma F_x = 0 \rightarrow T_{AC} = -(3/\sqrt{13})\cdot T_{AD} = \mathbf{+9\ kN}$ (tension); $T_{CB} = T_{AC}$ by symmetry

A zero-force member is a useful thing to include in a verification: it's a common source of solver bugs (a member whose stiffness contributes to the system but whose correct answer is exactly zero — easy to get subtly wrong, unlike a large nonzero force where a bug might be masked by "looking about right").

3. FEM model

Built directly against the calculator's core solver (`analyze()`), bypassing the UI: every member idealized as a pure axial (truss) element via `trussReleases()` (both bending moments released at both ends), a pin support at A, a roller at B, and out-of-plane bracing at the two free joints C and D to keep the truss planar. With every member's bending stiffness released, the joint rotational DOFs carry no stiffness at all — the solver's auto-constrain mechanism restrains them automatically rather than reporting a singular system.

4. Results comparison

Reactions

Quantity	Hand solution	FEM result	Difference
Reaction A_x	0.000000	3.55×10^{-15}	≈ 0
Reaction A_y	6.000000	5.999999999999993	7×10^{-15}
Reaction B_y	6.000000	5.999999999999993	7×10^{-15}

Member axial forces

Member	Hand solution [kN]	FEM result [kN]	Difference
AC (tie beam)	+9.000000	8.999999999999986	1.4×10^{-14}
CB (tie beam)	+9.000000	8.999999999999986	1.4×10^{-14}
AD (rafter)	−10.816654	−10.816653826391956	4×10^{-13}
DB (rafter)	−10.816654	−10.816653826391954	4×10^{-13}
CD (king post)	**0.000000**	**−0.000000**	≈ 0

✓ All quantities match to floating-point precision, including the zero-force member VERIFIED

5. Conclusion

The FEM calculator reproduces the exact method-of-joints solution for this king-post truss to within floating-point round-off (~10^{-12}–10^{-13}), for all support reactions and every member's axial force — including correctly identifying the zero-force king post, rather than reporting a small spurious value. This exercises a different code path than the earlier three-hinged-frame check: pure axial (truss) elements via member end-releases on both ends, rather than bending-dominated frame behavior.

Generated as part of an ad-hoc verification exercise — not a permanent addition to the automated test suite. The equivalent scripted check (built and run against `src/core` directly via Vitest) was deleted after use, per this session's convention for one-off scratch verification.